Student Number__________

MORIAH COLLEGE

Year 12 – Task 2 - Pre-Trial

MATHEMATICS
2014

Time Allowed: **3 hours**

Examiners: G. Busuttil, O. Golan,

OUTCOMES ADDRESSED: P3,P5,H2,H4,H5,H6,H7,H8

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the end of this paper
- All necessary calculations should be shown in every question.

Section I
Multiple choice questions 1-10
10 marks

Section II
Short response questions 11-16
90 marks

Total marks: 100

Section I Multiple Choice Questions. 1 mark each.

Circle the correct response on the answer sheet provided at the end of the paper.

(1) Factorise $2h^2-11h+15$

A. $(2h+5)(h+3)$ B. $(2h+5)(h-3)$ C. $(2h-5)(h+3)$ D. $(2h-5)(h-3)$

(2) How many terms are in the sequence $\sum_{7}^{30} 3n$

A. 21 B. 22 C. 23 D. 24

(3) The derivative of $y=\frac{4}{x^3}$ is

A. $y'=\frac{-12}{x^4}$ B. $y'=\frac{-12}{x^{-4}}$ C. $y'=4x^{-3}$ D. $y'=\frac{-2}{x^2}$

(4) Solve $|2x-1|\le 5$

A. $-2\ge x\ge 3$ B. $-2\le x\le 3$ C. $x\le -2, x\ge 3$ D. $x\le -3, x\ge 2$

(5) If $\log_t z=p$, then

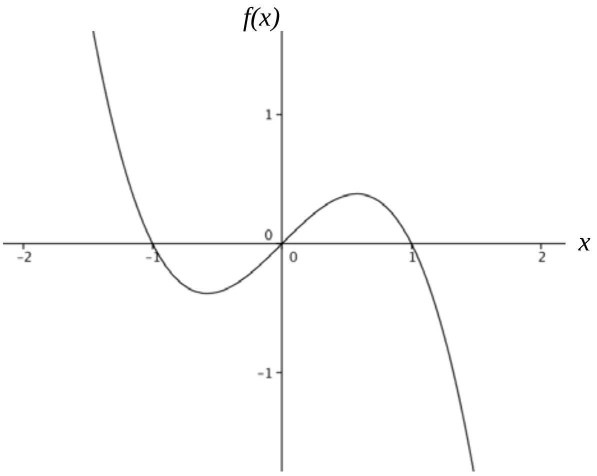
A. $z=p^t$ B. $p=z^t$ C. $z=t^p$ D. $p=t^z$

(6) Factorise a^3-64

A. $(a-4)(a^2-8a+16)$ B. $(a-4)(a^2+8a+16)$

C. $(a-4)(a^2-4a+16)$ D. $(a-4)(a^2+4a+16)$

Questions 7 and 8 both refer to the function $f(x) = -x^3 + x$



(7) *f(x)* is:

A. even
B. odd
C. neither
D. unable to be determined

(8) The integral of $f(x) = -x^3 + x$ from x = –1 to x = 1 is:

A. $2\int_{-1}^{0}(-x^3 + x)dx$ B. $2\int_{0}^{1}(-x^3 + x)dx$

C. Either A or B   D. 0

(9) The exact value of $\sin 225^o$ is:

A. $\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{2}}{2}$ D. $-\frac{\sqrt{2}}{2}$

(10) If $\log_c 2 = 0.46$, $\log_c 3 = 0.67$, $\log_c 5 = 1.27$, then $\log_c 30 = ?$

A. 0.391414   B. 2.4   C. 1.1591   D. 6.7

Section II Short response Questions. 15 marks each.

Question 11 (START A NEW BOOKLET)

(a) Calculate the perpendicular distance of the point (3, -1) from the line $4y = 3x + 2$. **2**

(b) Express $\dfrac{\log_3 8}{\log_3 2}$ as an integer **2**

(c) Evaluate $\displaystyle\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$ **2**

(d) Determine the value of n to make the following expression equal to a **single** digit number: **2**

$$5^2 \times 2^4 \times 10^{-n}$$

(e) Find the equation of the tangent to the curve $y = 5\log_e x$ at $x = 1$. **3**

(f) Solve for x: $(4x-3)^2 = 25$ **2**

(g) If $(3+\sqrt{3})^2 = a + b\sqrt{3}$, find the values of a and b. **2**

END OF QUESTION 11

Question 12 (START A NEW BOOKLET)

(a) Differentiate with respect to *x*:

(i) x^2e^{-x} **2**

(ii) $\dfrac{x^2}{3x+1}$ **2**

(b) Find $\int \dfrac{2x^2}{2x^3-3}dx$ **2**

(c) If α and β are the roots of the equation $3x^2-4x-1=0$, find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(ii) $\alpha^{-1}+\beta^{-1}$ **1**

(iv) $\alpha\beta^3+\beta\alpha^3$ **2**

(d) (i) Solve the equation $5^{3x}=0.04$ **2**

(ii) Solve $\log_2 x-\log_2(x-3)=2$ **2**

END OF QUESTION 12

Question 13 (START A NEW BOOKLET)

(a) The third term of an arithmetic progression is 23 and the tenth term is 72.

(i) Find the first term and the common difference. **2**

(ii) Calculate the sum of the first 18 terms. **2**

(b) The first term of a geometric progression is 6 and the common ratio is 3. **2**
How many terms of this progression are required to give a sum of 1594320?

(c) The derivative of a function is given by $f'(x) = 15(5x-1)^2$. **2**
If $f(0) = 10$, find the equation $f(x)$

(d) (i) Find the equation of the locus of $P(x, y)$, if P is always equidistant from $A(3, 1)$ and $B(1, 3)$. **2**

(ii) Give a geometric description of this locus. **1**

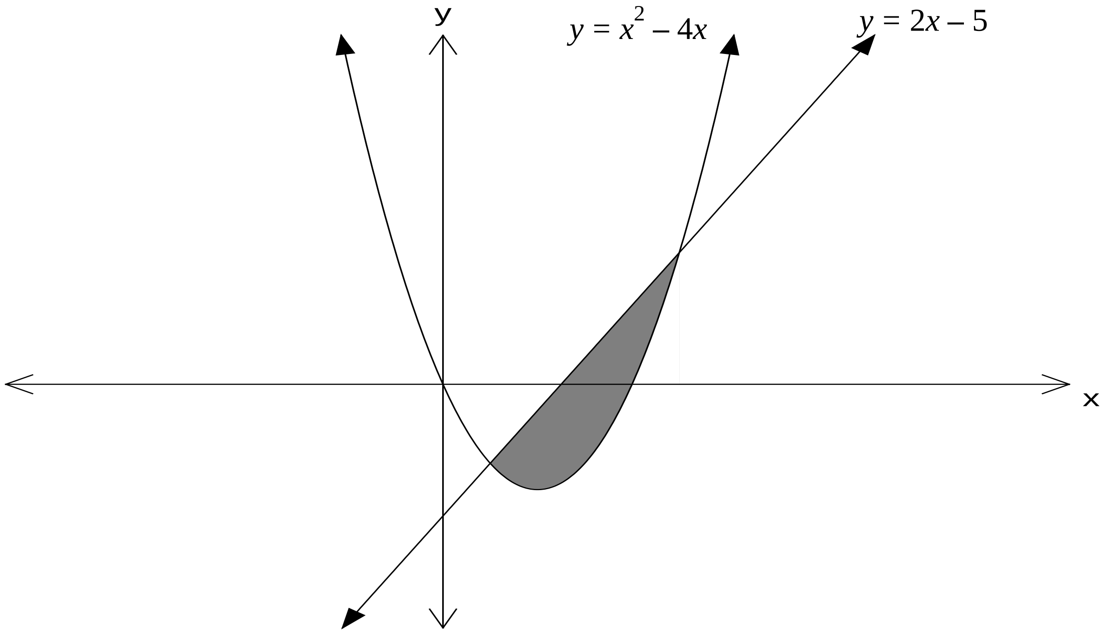
(e) (i) On the same set of axes, graph $y = |2x-1|$ and $y = -x$. **3**

(ii) Use your graph, or otherwise, to explain why $|2x-1| + x = 0$ has no solutions. **1**

END OF QUESTION 13

Question 14 (START A NEW BOOKLET)

(a) The graph shows the curves $y = x^2 - 4x$ and $y = 2x - 5$



(i) Show the curves intersect when $x = 1$ and $x = 5$. **2**

(ii) Find the shaded area between the two curves **3**

(b) Solve $\log_7 x^2 = 3$. **2**
Give your answer in exact simplified form.

(c) Consider the function $f(x) = x^3 + 6x^2 + 9x + 4$ in the domain $-4 \le x \le 1$

(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Determine the coordinates of its point(s) of inflexion. **2**

(iii) Draw a sketch of the curve $y = f(x)$ in the domain $-4 \le x \le 1$ clearly showing all its essential features. **2**

(iv) What is the global maximum value of the function $y = f(x)$ in the domain $-4 \le x \le 1$? **1**

END OF QUESTION 14

Question 15 (START A NEW BOOKLET)

(a) Tom sets a pendulum swinging and notices that each swing is 80% as long as the preceding swing. The first swing is 20cm, the second swing is 16cm, and it continues to swing until coming to rest. **2**

What is the total distance the pendulum swings?

(b) Find the focus and directrix of the parabola $x^2 - 8x - 16y + 48 = 0$ **2**

(c) Prove that $\dfrac{\tan\theta\sec\theta}{1+\tan^2\theta} = \sin\theta$. **2**

(d) (i) Differentiate $y = xe^x$ **1**

(ii) Hence, evaluate $\displaystyle\int_0^2 \frac{xe^x}{2}\, dx$ **3**

(e) (i) Use the trapezoidal rule with 5 function values to find an approximation to **3**

$$\int_0^2 \frac{1}{x+1}\, dx$$

(ii) Calculate the difference between your answer in part (i) to the exact value, correct to 3 decimal places. **2**

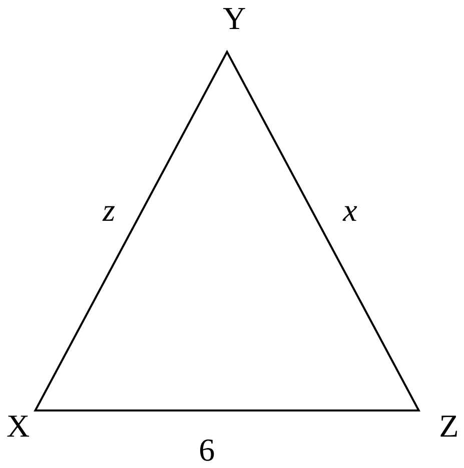
END OF QUESTION 15

Question 16 (START A NEW BOOKLET)

(a) Find the maximum value of the function $y = -16x^2 + 160x - 256$ **2**

(b) Triangle XYZ has $XZ = 6$, $YZ = x$ and $XY = z$, as shown.

The perimeter of ΔXYZ is 16. All measurements are in centimetres.



(i) Express z in terms of x **1**

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$ **1**

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$ **2**

(iv) Let the area of ΔXYZ be A. **1**
Show $A^2 = 9x^2 \sin^2 Z$

(v) Hence, show that $A^2 = -16x^2 + 160x - 256$ **2**

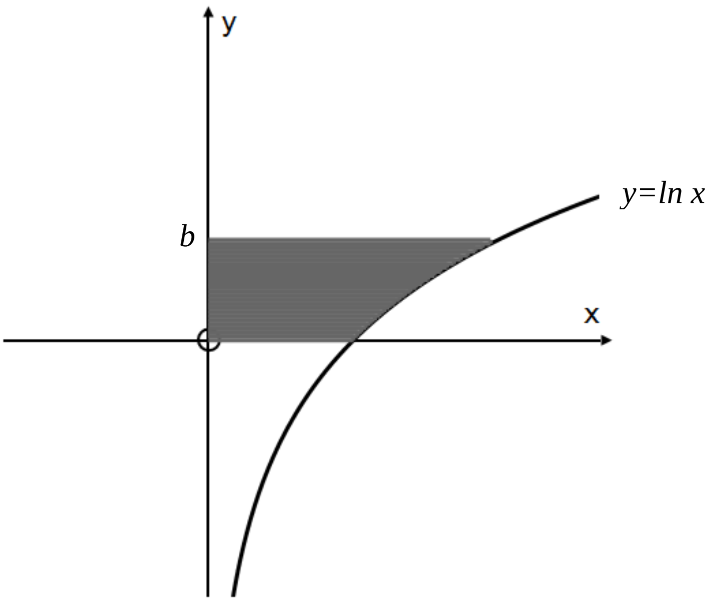
(vi) Using your answer from question (a), or otherwise, find the maximum area for ΔXYZ? **1**

Question 16 continues on the next page

Question 16 (Continued)

(b) The quadratic equation $(k+1)x^2 - 4kx + 4k - 3 = 0$ has a root equal to 1. **2**
Find *k*.

(c) The shaded area is one square unit. Find the exact value of *b*. **3**



END OF EXAM

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Student Number: ....

Teacher: ....

<u>CIRCLE EACH CORRECT ANSWER</u>.

MULTIPLE CHOICE ANSWER SHEET

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

PRETRIAL MATHEMATICS
2014

SECTION 1 MULTIPLE CHOICE

1. $2h^2 - 11h + 15$
   $(2h-5)(h-3)$ ... (D)

2. $30 - 7 + 1 = 24$ ... (D)

3. $y = 4x^{-3}$
   $y' = -12x^{-4}$
   $= \frac{-12}{x^4}$ ... (A)

4. $|2x-1| \le 5$
   $-5 \le 2x-1 \le 5$
   $-4 \le 2x \le 6$
   $-2 \le x \le 3$ ... (B)

5. $\log_t z = p$ then
   $z = t^p$ (C)

6. $a^3 - 64$
   $= (a-4)(a^2+4a+16)$ ... (D)

7. (B) odd

8. (D)

9. sin 225 ... (D)

10. $\log_c 30 = \log_c (2 \times 3 \times 5) = 0.46 + 0.67 + 1.27 = 2.4$ (B)

1	D
2	D
3	A
4	B
5	C
6	D
7	B
8	D
9	D
10	B

SECTION II

Question 11

a) $d = \left| \dfrac{3\times3 - 4(-1) + 2}{\sqrt{3^2+4^2}} \right|$ (1)

$= \dfrac{9+4+2}{5}$

$= \dfrac{15}{5}$

$= \boxed{3}$ (2)

b) $\dfrac{\log_3 8}{\log_3 2} = \dfrac{\log_3 2^3}{\log_3 2}$ (1)

$= \boxed{3}$ (2)

c) $\lim\limits_{x\to4} \dfrac{(x-4)(x+4)}{(x-4)}$ (1) $= \boxed{8}$ (2)

d) $5^2 \times 2^4 \times 10^{-n} = \dfrac{25\times16}{10^n}$

$= \dfrac{400}{10^n}$ (1) or $10^n = 100$

$\dfrac{400}{10^n} < 10$ (single digit)

(2) $\boxed{n=2.}$ $\dfrac{400}{10^2} = \dfrac{400}{100} = 4.$

e) $y' = \dfrac{5}{x}$ (1) $x=1,\ m=5,\ y=0.$ (2)

$y - 0 = 5(x-1)$ (3)

$\boxed{y = 5x-5}$

f) $(4x-3)^2 = 25$

$4x-3 = \pm5$

$4x-3=5 \qquad 4x-3=-5$

$4x=8 \qquad 4x=-2$

$\boxed{x=2 \qquad x=-\tfrac{1}{2}}$

g) $(3+\sqrt3)^2 = a+b\sqrt3$

$9+6\sqrt3+3 = a+b\sqrt3$

$12+6\sqrt3 = a+b\sqrt3$

$\boxed{a=12 \quad b=6.}$

ECT = error carried thru.

Question 12.

a) (i) $y = x^2e^{-x}$ ✓ Product rule

$y' = 2x.e^{-x} - x^2.e^{-x}$ ✓

$= xe^{-x}(2-x)$

(ii) $y = \dfrac{x^2}{3x+1}$

$y' = \dfrac{(3x+1)(2x) - x^2(3)}{(3x+1)^2}$ ✓

$= \dfrac{6x^2+2x-3x^2}{(3x+1)^2}$

$= \dfrac{3x^2+2x}{(3x+1)^2}$ ✓

b) $\displaystyle\int \frac{2x^2}{2x^3-3}\,dx$

$= \dfrac{1}{3}\displaystyle\int\frac{6x^2}{2x^3-3}\,dx$ ✓

$= \frac{1}{3}\ln(2x^3-3) + C$ ✓

c) i) $\alpha+\beta = \frac{4}{3}$ ✓

ii) $\alpha\beta = -\frac{1}{3}$ ✓

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta}$

$= \dfrac{\frac{4}{3}}{-\frac{1}{3}}$ ✓

$= -4.$

(iv) $\alpha\beta^3 + \beta\alpha^3 = \alpha\beta(\beta^2+\alpha^2)$

$= \alpha\beta[(\alpha+\beta)^2 - 2\alpha\beta]$ ✓

$= -\frac{1}{3}[(\frac{4}{3})^2 - 2(-\frac{1}{3})]$

$= -\frac{16}{27} - \frac{2}{9} = \frac{-22}{27}$ ✓

d) (i) $5^{3x} = 0.04$

$\log_5 0.04 = 3x$ ✓

$x = \dfrac{\log_5 0.04}{3}$

$= \dfrac{\log_5 5^{-2}}{3}$

$= \dfrac{-2.\log_5 5}{3}$

$= \dfrac{-2}{3}.$ ✓

(ii) ~~$\log_2 x(x-3) = 2.$~~

~~$x(x-3) = 2^2$~~

~~$x^2 - 3x - 4 = 0$~~

~~$(x-4)(x+1) = 0$~~

~~$x = 4, -1$~~

$\log_2\left(\dfrac{x}{x-3}\right) = 2.$ ✓ ✓

$\dfrac{x}{x-3} = 2^2$

$x = 4x-12$

$3x = 12$

$x = 4$ ✓

Question 13.

a) i) T_3: $a+2d=23$

T_{10}: $a+9d=72$

$7d=49$

$d=7$ 1

$\therefore a=9$ 1

(2)

ii) $S_{18}=\frac{18}{2}[2\times9+(17\times7)]$ 1

$=1233$ 1

(2)

b) $a=6$ $r=3$.

$S_n=1594320$

$\frac{6(3^n-1)}{3-1}=1594320$ 1

$3^n-1=\frac{1594320}{3}$

(2)

$3^n=531441$

$\ln 3^n=\ln 531441$

$n=\frac{\ln 531441}{\ln 3}$

$n=12.$ 1

c) $f'(x)=15(5x-1)^2$

$f(x)=\frac{15(5x-1)^3}{5\times3}+c$

$=(5x-1)^3+c$ 1

$f(0)=10$, $10=(-1)^3+c$

$10=-1+c$

(2)

$c=11$

$f(x)=(5x-1)^3+11$ 1

$P(x,y)$ $A(3,1)$ $B(1,3)$

d) $PA^2=PB^2$

(i) $(x-3)^2+(y-1)^2=(x-1)^2+(y-3)^2$ 1

$x^2-6x+9+y^2-2y+1=x^2-2x+1+y^2-6y+9$

$-6x-2y+10=-2x-6y+10.$

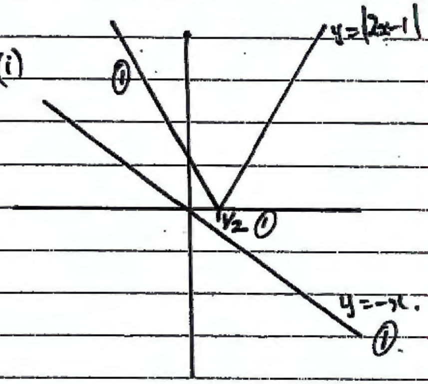
$-4x+4y=0.$

(2)

$y=x.$ 1

(ii) Straight line with gradient +1, through (0,0) (1)

e) (i)



(3)

(ii) $|2x+1|=-x$ 1

$|2x+1|+x=0$

(1)

No solutions since the graphs do not intersect.

Question 14

(a). (i) $x^2 - 4x = 2x - 5$. ✓

$x^2 - 6x + 5 = 0$ 2

$(x-5)(x-1) = 0$ ✓

$x = 1, 5$.

ii) Area $= \int_1^5 \left[(2x-5) - (x^2 - 4x)\right] dx$.

$= \int_1^5 (-x^2 + 6x - 5)\, dx$ ✓

$= \left[\frac{-x^3}{3} + 3x^2 - 5x\right]_1^5$ ✓

$= \left(\frac{-125}{3} + 75 - 25\right) - \left(\frac{-1}{3} + 3 - 5\right)$

$= 8\frac{1}{3} + 2\frac{1}{3}$

$= \frac{32}{3} / 10\frac{2}{3}$ units^2 ✓   3.

b) $\log_7 x^2 = 3$.

$x^2 = 7^3$ ✓   2

$x = \sqrt{7^3}$ ✓

$x = \pm\sqrt{343}$

c) $f(x) = x^3 + 6x^2 + 9x + 4$ $-4 \le x \le 1$

(i) $f'(x) = 3x^2 + 12x + 9$ 3.

SP $f'(x) = 0$ $x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -1, -3$.

$f''(x) = 6x + 12$ ✓

$x = -1$, $f''(x) = -6 + 12 > 0$ ∴ min. $(-1, 0)$ min. ✓

$x = -3$, $f''(x) = -18 + 12 < 0$ ∴ max. $(-3, 4)$ max. ✓

ii) Possible P.O.I. $f''(x) = 0$.

$6x + 12 = 0$.

$x = -2$.

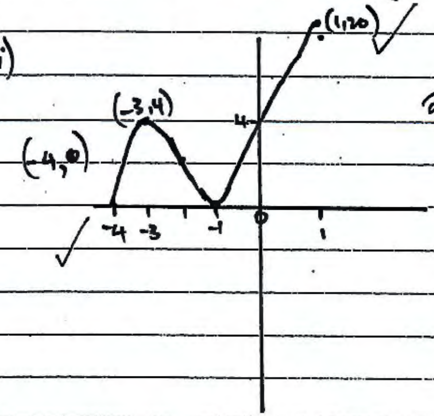
x	-3	-2	-1
$f''(x)$	$-$	0	$+$

✓   2.

Change in concavity ✓

∴ P.O.I. $(-2, 2)$

iii)



$-4 \le x \le 1$ 2.

(iv) global max in this domain $y = 20$. ✓   1.

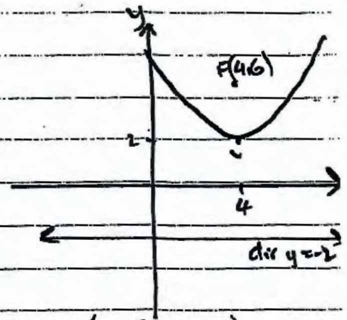
15

Question 15.

a) $a = 20$
$r = 0.8$ ✓

$S_\infty = \frac{20}{1-0.8}$ ✓
$= 100$ cm

b) $x^2 - 8x - 16y + 48 = 0$.
$x^2 - 8x + 16 = 16y + 16 - 48$
$(x-4)^2 = 16y - 32$
$(x-4)^2 = 16(y-2)$ ✓
$\therefore$ Vertex $(4,2)$
$a = 4$
Focus $(4,6)$ ✓
Directrix $y = -2$.



no mark if one is wrong.

c) $LHS = \frac{\tan\theta . \sec\theta}{1+\tan^2\theta}$ $RHS = \sin\theta$

$= \frac{\tan\theta . \sec\theta}{\sec^2\theta}$ ✓

$= \frac{\tan\theta}{\sec\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \cos\theta$ ✓

$= \sin\theta$

$= RHS$

(d) (i) $y = x . e^x$
$y' = x(e^x) + e^x(1)$
$= xe^x + e^x$ ✓

(ii) $\int_0^2 \frac{xe^x}{2}\,dx$

$= \frac{1}{2}\left[xe^x - e^x\right]_0^2$ ✓

$= \frac{1}{2}\left[(2e^2 - e^2) - (0-1)\right]$ ✓   one mark deducted for incorrect coefficient

$= \frac{1}{2}(e^2+1)$ ✓

e) $\int_0^2 \frac{1}{x+1}\,dx$ 5 fn values   Trapezoidal Rule.

(i)

x	y	k	ky
0	1	1	1
$\frac{1}{2}$	$\frac{2}{3}$	2	$\frac{4}{3}$
1	$\frac{1}{2}$	2	1
$1\frac{1}{2}$	$\frac{2}{5}$	2	$\frac{4}{5}$
2	$\frac{1}{3}$	1	$\frac{1}{3}$
	✓	✓	$\frac{67}{15}$

$A \doteqdot \frac{\frac{1}{2}}{2}\left(\frac{67}{15}\right)$ ✓

$= \frac{67}{60}$.

(ii) $A = \int_0^2 \frac{1}{x+1}\,dx$

$= \left[\ln(x+1)\right]_0^2$

$= \ln 3 - \ln 1$ ✓

$= (\ln 3)$ units^2

Diff $= \frac{67}{60} - \ln 3$ ✓

$= 0.018$ (3 dp)

Question 16.

a) $y = -16x^2 + 160x - 256$

$y' = -32x + 160$

SP when $y' = 0$ $\quad 160 = 32x$

$x = 5$. 1

$y'' = -32 < 0 \therefore$ max. $\quad x = 5,\ y = 144$

Max y value is 144. 1

(2)

b) (i) $x + z + 6 = 16$

$x + z = 10$.

$z = 10 - x$. 1

(1)

(ii) $z^2 = x^2 + 36 - 12x.\cos Z$. (1)

(1)

~~$z^2 = (10-x)^2 + 36 - 12x.\cos Z$~~

~~$z^2 = 100 - 20x + x^2 + 36 + 12x\cos Z$.~~

(iii) $(10-x)^2 = x^2 + 36 - 12x.\cos Z$. 1

$100 - 20x + x^2 = x^2 + 36 - 12x.\cos Z$

$64 - 20x = -12x\cos Z$.

$12x\cos Z = 20x - 64$

$\cos Z = \dfrac{20x - 64}{12x}$

$\cos Z = \dfrac{5x - 16}{3x}$ 1

(2)

iv) $A = \frac{1}{2} \times 6x.\sin Z$

$A = 3x.\sin Z$

$\therefore A^2 = 9x^2.\sin^2 Z$. 1

(1)

v) $A^2 = 9x^2(1 - \cos^2 Z)$ 1

$= 9x^2\left(1 - \left(\dfrac{5x-16}{3x}\right)^2\right)$

$= 9x^2 - (5x-16)^2$

$= 9x^2 - (25x^2 - 160x + 256)$

$A^2 = -16x^2 + 160x - 256$ 1

(2)

vi) $(A^2) = -16x^2 + 160x - 256$

from part (a), max value of $A^2 = 144$

$\therefore$ Max Value of $A = 12$ units

(1)

(c) $(k+1)x^2 - 4kx + 4k - 3$ $\quad$ let roots be 1 and β.

Sum: $\alpha + \beta = \dfrac{+(4k)}{k+1}$ $\qquad$ Prod $\alpha\beta = \dfrac{4k-3}{k+1}$

$1 + \beta = \dfrac{4k}{k+1}$ $\qquad$ $\beta = \dfrac{4k-3}{k+1}$ 1

$\beta = \dfrac{4k}{k+1} - 1$

$\therefore \dfrac{4k}{k+1} - 1 = \dfrac{4k-3}{k+1}$

$4k - (k+1) = 4k - 3$

$4k - k - 1 = 4k - 3$

$-k = -2$

$k = 2$. 1

(2)

(d) $y = \log_e x$ Area $= 1$

$\therefore x = e^y$ 1

Area $= \int_0^b e^y \, dy = 1$

$\left[e^y\right]_0^b = 1$ 1.

$e^b - e^0 = 1$

$e^b - 1 = 1$

$e^b = 2$ 1.

$\ln e^b = \ln 2$

$b = \ln 2.$ 1

3